<u>Section 1.1</u>

In math, we use **<u>variables</u>** to talk about

[1]

OR [2]

ex. <u>Rewrite using variables. Which of the two cases above is each situation ?</u>

"Everyone enrolled in Math 22 passed Math 43."

"Some real numbers are smaller than their own square roots."

<u>Rewrite without using variables. (Try to make the answer sound like natural language.)</u>

"There is a DeAnza instructor r, such that r's wife is a chef."

"For all positive integers t, $\frac{1}{t} \leq t^2$."

A **<u>universal statement</u>** says that

Universal statements often use the words

or the phrases

A **<u>conditional statement</u>** says that

Conditional statements often use the words

An **<u>existential statement</u>** says that

Existential statements often use the word

or the phrases

ex. <u>Classify each statement.</u>

"Somebody in this class hasn't signed in yet."

"All DeAnza students have a DeAnza ID number."

"If 97 is odd, then 97^2 is odd."

A **<u>universal conditional statement</u>** says that

In other words, a universal conditional statement is a statement that is

eg. "For all American citizens p, if p is eligible to vote, then p is at least 18 years old."

Universal conditional statements can be written to appear as either strictly universal or strictly conditional.

eg. Universal: "For all American citizens w who are eligible to vote, w is at least 18 years old."

OR "All American citizens who are eligible to vote are at least 18 years old."

Conditional: "If an American citizen w is eligible to vote, then w is at least 18 years old."

OR "If an American citizen is eligible to vote, he is at least 18 years old."

NEW GROUP = THINGS IN ORIGINAL GROUP FOR WHICH THE "IF" PROPERTY IS TRUE

NEW "IF" PROPERTY = ORIGINAL "IF" PROPERTY ALONG WITH PROPERTY OF BEING IN ORIGINAL GROUP

Universal conditional statements can also be written to appear neither explicitly universal nor explicitly conditional.

eg. "American citizens must be 18 years old to be eligible to vote."

SIDE NOTE: It is possible to write every universal statement as a conditional statement using the method above.

ex. <u>Rewrite using the given structures.</u>

"For all real numbers m, if $m < 0$, then $\sqrt{m}$ is an imaginary number."

USING A VARIABLE:

[a] If m , then

[b] For all m,

WITHOUT USING A VARIABLE:

[c] All

[d] The square root of

[e] If , then

ex. <u>Rewrite using the formal universal conditional structure.</u>

"The sine of every acute angle is positive."

For all , if , then

A **<u>universal existential statement</u>** says that

eg. “For every positive number *x*, there is an acute angle *y* such that $y = \tan^{-1} x$.”

Universal existential statements can be written in a less formal structure,
which may make the existential portion less explicit by eliminating the second variable or even both variables.

eg. “For all positive numbers *x*, *x* has an acute inverse tangent.”

“Every positive number has an acute inverse tangent.”

“All positive numbers have acute inverse tangents.”

“The inverse tangent of a positive number is always acute.”

ex. <u>Rewrite using the given structures.</u>

“For all negative numbers j, there is a positive number k, such that $k = j^2$.”

USING ONE VARIABLE:

[a] For all j, j has

[b] For every , there is

WITHOUT USING A VARIABLE:

[c] All

[d] The square of

[e] For every , there is

ex. <u>Rewrite using the formal universal existential structure.</u>

“Everybody loves somebody.”

For every , there is , such that

An **existential universal statement** says that

eg. “There is a positive number e such that, for all real numbers r, $e \times r = r$.”

Existential universal statements can be written in a less formal structure,
by eliminating the second variable or even both variables.
It is, however, hard to make the existential or universal portions less explicit.

eg. “There is a positive number e whose product with any real number is that real number.”

“There is a positive number whose product with every real number is the real number.”

“Some positive number, when multiplied by any real number, gives that real number.”

ex. Rewrite using the given structures.

“There is a class g such that, for every Math 22 student b, b has passed g.”

USING ONE VARIABLE:

[a] There is a class g such that

WITHOUT USING A VARIABLE:

[b] There is

[c] Some

ex. Rewrite using the formal existential universal structure.

“Some monument has been seen by every American tourist visiting Paris.”

There is such that, for all ,

Section 1.2

A set is a collection or group of elements.

eg. if M = set of all Honda car models

then Fit M ie.

and Prius M ie.

Set roster notation (list of elements)

eg. set of factors of 8 = {1, 2, 4, 8}

set of integers from 5 to 20 =

set of integers greater than 5 =

Special sets

= set of all real numbers

= set of all integers

= set of all rational numbers
(quotients of integers)

= set of all *positive* real numbers

= set of all *negative* integers

= set of all *non-negative* rational numbers
(zero and all positive rational numbers)

Set equality

INFORMAL DEFINITION:

Given sets A and B, we say A and B are equal, or A = B,

ex. If A = {1, 2, 3} and B = {3, 1, 2}, then A B

If C = {0, 2, 4, 6} and D = {2, 4, 6}, then C D

A set can be an element of another set.

eg. Let K = {a, {b}}

a K {b} K {a} K b K

Set builder notation (specification of property)

A limitation of set roster notation is that for sets with many elements,
you must either list all the elements,which would be impossible for sets with infinitely many elements,
or you must use ellipsis, but the pattern of the elements may not be obvious

eg. {3, 4, 6, 8, 12, 14, …}

Given a set S, and a property P which may or may not be true for the individual elements of S, we can define a new set

which consists of exactly those elements of S for which P is true,
ie. those elements of S which *satisfy* P.

eg. $\{x \in \mathbf{Z}^+ \mid -3 \le x < 3\} =$

$\{x \in \mathbf{Z} \mid x = 6k \text{ for some integer } k\} =$

set of all positive integers which are 1 larger than a prime number =

set of all perfect squares =

<u>Subsets</u>

DEFINITION:

Given sets A and B, we say A is a subset of B, or

if and only if

Written more casually, if and only if

Other ways of reading $A \subseteq B$:

NOTE: A is not a subset of B, or

if and only if

(or more symbolically)

ex. $\{1, 2\}$ $\{0, 1, 2, 3\}$ $\{0, 1, 2, 3\}$ $\{1, 2\}$

$\{1, 2, 4, 8\}$ $\{0, 1, 2, 3, 4, 5, 6, 7\}$ $\{x \in \mathbf{Z}^+ \mid x \text{ is prime}\}$ $\{x \in \mathbf{Z}^+ \mid x \text{ is odd}\}$

$\{4, 7\}$ $\{4, 7\}$ 1 $\{1, 2\}$

$\{2\}$ $\{1, \{2\}\}$

<u>Ordered Pairs; Cartesian Product of 2 Sets</u>

DEFINITION:

Given elements a, b, c, d, we say (a, b) = (c, d) if and only if

eg. (1, 4) (4, 1)

(0, 2) $(\sin \pi, \sqrt{4})$

We can think of ordered pairs as special sets,
where the ordered pair (a, b) corresponds to the set {{a}, {a, b}}.

eg. (1, 4) corresponds to the set

(4, 1) corresponds to the set

(2, 2) corresponds to the set

{{2, 5}, {5}} corresponds to the ordered pair

DEFINITION:

Given two sets A and B, the Cartesian product of A and B, or is

Written more casually, $A \times B$ is the set of all ordered pairs where

eg. Given A = {1, 3} and B = {1, 2}

$A \times B =$

$B \times A =$

ex. Given R = {0, 4} and T = {a, g, r}

$R \times R =$

$T \times R =$

The number of elements in the Cartesian product of 2 sets is

ex. Given $Q \times P$ = {(a, t), (h, h), (h, e), (a, e), (a, h), (h, t)}

P =

Q =